



## Multilevel Models: Introduction and Applications

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## 2 Goals for this talk

- When to use multilevel models?
  - Conceptual understanding
- Links with other methods: regression, anova
- Examples
- Software choices



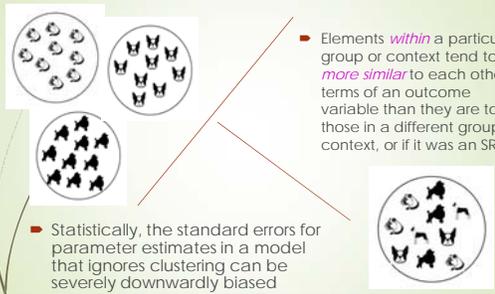


## 3 What are multilevel models (MLMs)?

Models that capture the structure of data from clustered, nested, or hierarchical samples, such as:

- Children within families (2 levels)
- Students within classrooms or schools (2 levels)
- Clients within programs within community agencies (3 levels)
- Occasions within persons (repeated measures)
- Residents within communities repeated over time (community trials)
- Subjects within studies (meta-analysis)

## 4 Homogeneity of clusters



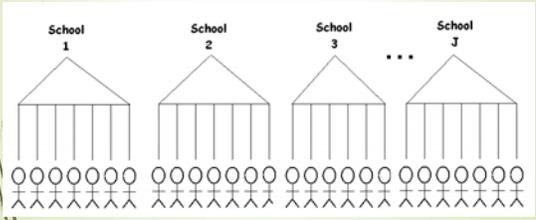
- Elements *within* a particular group or context tend to be *more similar* to each other in terms of an outcome variable than they are to those in a different group or context, or if it was an SRS.
- Statistically, the standard errors for parameter estimates in a model that ignores clustering can be severely downwardly biased
  - Too small, every effect is statistically significant!

## 5 The Nesting Problem

- Homogeneity is measured as a correlation or dependency (ICC or  $\rho = \rho$ ) in the data
  - Intraclass correlation coefficient
  - Tells us how much of the variability in the outcome of interest lies between groups
  - Measures the correlation between two randomly selected units from the same cluster
- The existence of ICC *violates the assumption of independence* required for a traditional linear model's approach to data analysis.
- Even mild violations can lead to severe problems with Type I error (typically inflated).

## 6 Two-level structure

Students, nested within J schools



**7** Three-level structure

When schools belong to (one of) K different districts:

**8** Cross-classified structure

*Students, cross-classified by school and neighborhood*  
Beretvas, S. N. (2008)

**9** Repeated measures structure

*Data collected over months; some participants provide only partial data to the model.*  
*This is an example of "time-unstructured" data, since collection schedules are variable across participants.*

**10** Importance of Context

- Contexts can be considered as replications of one another, sampled from a single population of contexts
  - Schools
  - Neighborhoods
  - Hospitals
  - Community clinics or organizations
- Are relationships within each setting the same?
  - Does a treatment or intervention work similarly across settings? If not, why are they different? Are relationships influenced by characteristics of the group or setting? How can these differences be captured?

**11** Multilevel Research Questions

- Use of multilevel data can clarify how *person-level variables* (e.g., gender, behavior, attitudes, knowledge, general health) and *cluster-level variables* (e.g., characteristics of classrooms or teachers, schools, neighborhoods, health-clinics, etc.) help to **explain an outcome of interest** (measured at the lowest level of the hierarchy)
- There can also be **interactions** that occur within levels and also **between-levels**
  - Cross-level interactions: between an individual-level variable and a context-level variable

**12** Cross-level Interactions

- Characteristics of contexts or settings may contribute to individual outcomes
  - Within contexts or groups, **relationships may vary** depending on (conditional on) group characteristics
    - Cross-level interactions
    - Effect of a lower-level variable moderated by a higher-level variable

13 Graphical Example (R&B, 2002)

- High School and Beyond (HSB)
  - Effect of SES on Math Achievement
- 7165 students nested within 160 schools
- Q1: Does family SES **predict** students' academic achievement in math?
- Q2: Are there differences across schools (i.e., is there **variability**) in the relationship between SES and math achievement?
- Q3: If yes, are there **school-level factors** that might help to explain this variability?

14 10 schools

$$\hat{Y} = b_0 + b_1SES$$

15 ... Now Identified by Sector

$$\hat{Y} = b_0 + b_1SES$$

Public are blue (0)  
Private are red (1)

16 Organizational Contexts

- In HSB, we see that intercepts can vary and slopes can vary between schools.
- Research questions often focus on how variables at level 2 contribute to these differences (and relatedly, the outcome of interest)
- These simple graphs show how **context** – here, the public versus private school distinction – can be important to understanding **differences in the relationships** between SES and Math Achievement

17 Longitudinal Research

- Each *person* in a longitudinal research study can be thought of as a distinct “group” or context
- Outcome and occasions are level-1 variables
- Person characteristics (gender, or intervention group, etc.) may be level-2 variables

18 It's all about variability in relationships...

- Do group level variables **moderate** the effect of an independent variable on Y?
  - Conditional Models
    - Cross-level interactions**, representing interactions between level-one and level-two variables
    - Micro to Macro level
- HLM provides a methodology for **connecting** the two (or three, etc.) levels together.

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## Research Examples

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## Neighborhood Context and Cognitive Decline (1)

- How is neighborhood context related to cognitive function of older Mexican Americans?
- Hispanic EPESE Longitudinal study; data linked with US Census; n = 3050 MA's in 208 nbhoods, followed over 5 years
  - Neighborhood variables included SES (economic disadvantage) and %MA (social disadvantage)
- three multilevel analyses
  - HLM 2-level just using baseline data and examining impact of nbhood variables on baseline cognitive functioning.
  - HLM 3-level (time, person, nbhood) individual growth curves for cognitive function over three waves
  - HGLM 2-level analysis (person, nbhood) for cognitive decline (yes or no) between time 1 and time 3
- Within and between-neighborhood variability examined (Sheffield & Peek, 2009)

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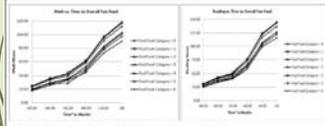
## Neighborhood Context and Cognitive Decline (2)

- 16% of the variability in functional decline is between neighborhoods
- Even after controlling for individual-level predictors, cognitive decline was faster among MA's in lowSES nbhoods; Odds of decline increased as percent of MA's in nbhood decreased (social disadvantage), and increased as economic disadvantage increased.
- Examined attrition and sensitivity analyses to bolster strength of findings
- For Mexican Americans, **neighborhood context matters** in reducing or slowing cognitive decline. (Sheffield & Peek, 2009)

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## Obesity, high-calorie food intake, and achievement (1)

- What school and individual/family characteristics including diet quality (consumption and access) are associated with students' academic achievement?
- Retrospective growth, ECLS-K math and reading achievement K through 5<sup>th</sup> grade; 6,178 children in 773 schools
- Self-report nutritional supplement in Grade 5
- We fit 3-level linear growth models for each set of longitudinal math and reading scores



- Trend-lines by fast-food consumption
- Top line is for no reported fast-food consumption

(Li & O'Connell, 2012)

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## Obesity, high-calorie food intake, and achievement (2)

- Even after adjusting for children's BMI status, more frequent high-calorie food intake (fast-food) was linked with children's **weaker academic achievement trends** (reading and math)
  - Lower achievement levels at Spring Grade 5
  - Weaker slopes across first six years of school
- Presence of vending machines on school property was not related to children's body weight over time, or to their academic achievement patterns
- Results are controlling for individual and school SES.
- Not causal, but clearly suggesting important trends between diet and cognitive development among US children (Li & O'Connell, 2012)

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## Stroke and fracture outcomes in US rehab facilities (1)

- Is there a relationship between facility volume and patient-centered outcomes among inpatient rehabilitation centers?
- Services in 717 rehabilitation facilities for 3 diagnostic groups: stroke (n = 202,423), lower-extremity fracture (n = 132,194), and lower extremity joint replacement (n = 148,068)
  - Quintiles for patient-volume served
- HLM: **continuous DV** = discharge functional status measure
- HGLM: **dichotomous DV** = probability of home discharge (1,0)
- Analyses controlled for clinical and sociodemographic variables
- US Data - UDSMR database (Uniform Data System for Medical Rehabilitation) (Graham, Deutsch, O'Connell, et al., 2013)

**25** Stroke and fracture outcomes in US rehab facilities (1)

Selected findings:

- Higher volume quintiles admit patients with lower functional status compared to lower volume quintiles
- Length of stay increased with greater volume for all 3 diagnostic groups
- % discharged home decreased with increasing volume in the fracture group: slight decrease for stroke, slight increase for joint replacement group
- Higher-volume stroke and fracture facilities tend to admit more clinically complex patients, leading to poorer predicted outcomes
- We were able to identify relatively low performing and high performing facilities

(Graham, Deutsch, O'Connell, et al., 2013)

**26** HLM Analysis Approach:

*Examine differences across clusters or higher-level units by assessing effects at multiple levels simultaneously through a level-1 and a level-2 (and, if needed, a level-3) model!*

**27** Analysis Considerations

- Partitioning Variance (ICC)
- Assumptions
- Model specification and Model interpretation
- Scaling – or re-centering – of predictors
- Variance accounted for
- Estimation methods
- Model fit
- Software
- New advances

**28** Illustration of HLM

- Substantive example using the ECLS-K
- Reading assessment (early literacy) for First Graders at the end of first grade.
  - 2408 students
  - 169 schools

DV = reading score  
Mean 57.77  
SD 12.94

**29** Student Level Descriptives

*Student Level Descriptive Statistics for the Analytic Sample,*  
N = 2408

Variable	M (SD)	57.77 (12.94)
Reading	M (SD)	57.77 (12.94)
Female	%	50.1%
NumRisks	M (SD)	.52 (.81)
Family SES	M (SD)	.10 (.74)

Note: reading variance is 167.26

**30** School Level Variables

*School Level Descriptive Statistics for the Analytic Sample, J = 169*

Variable	Mean	SD	Minimum	Maximum
Neighborhood Problems (NBHOODCLIM)	2.05	2.74	0.00	12.00
School Average SES	.05	.54	-1.37	1.40
% Private	21.3%	---	---	---

Private = 1, Public = 0

**31** HLM Variable Terminology

- $N = 2408$  students, nested within  $J = 169$  schools (approx. 14 students each school).
- $Y_{ij}$  = reading achievement for  $i$ th student in  $j$ th school
- Student-level** predictors of math achievement
  - $X_{1i}$  = Student (family) SES
  - $X_{2ij}$  = Numrisks
  - $X_{3ij}$  = Female
- School-level** predictors
  - $W_{1j}$  = MeanSES (SES averaged for each school)
  - $W_{2j}$  = NBHood Climate
  - $W_{3j}$  = Public or Private (School Sector)

**32** Partitioning the Variance (1)

- How much of the variability in reading achievement lies between schools?
- ICC = (variance between groups)/(total var)
- Assumption: Collection of schools' reading scores can be summarized as:
  - Each school has its own mean reading score,  $\beta_{0j}$
  - All schools share a common variance,  $\sigma^2$ , this represents dispersion in reading scores *within* the schools.
  - Global, or overall, summary of mean reading scores,  $\gamma_{00}$
  - Variation *between* schools is represented by  $\tau_{00}$

**33** One-way ANOVA with Random Effects

Level 1:  $Y_{ij} = \beta_{0j} + r_{ij}$

Level 2:  $\beta_{0j} = \gamma_{00} + u_{0j}$

**Combined Model**

$$Y_{ij} = \gamma_{00} + u_{0j} + r_{ij}$$

**Assumptions**

$$r_{ij} \sim N_{iid}(0, \sigma^2), \quad u_{0j} \sim N(0, \tau_{00}) \quad \text{and} \quad \text{cov}(r_{ij}, u_{0j}) = 0$$

**34** Results: Empty Model

	Coefficient (SE)	t (df)
<b>Fixed Effects</b>		
Model for the Intercepts ( $\beta_0$ )		
Intercept ( $\gamma_{00}$ ) (Mean Reading Achievement)	57.23 (.55)	104.55 (168)**
<b>Random Effects (Var. Components)</b>		
		Variance
Intercept ( $\tau_{00}$ )		40.14 **
Level-1 ( $\sigma^2$ )		129.77

**35** Intraclass Correlation

$$ICC = \frac{\text{between}}{\text{between} + \text{within}} = \frac{\tau_{00}}{\tau_{00} + \sigma^2} = \frac{40.14}{40.14 + 129.77} = .236$$

- 23.6% of the variability in reading achievement at the end of first-grade is at the school-level (between-schools)
- This also represents the correlation between two randomly selected children from the same school.

**36** Let's try a prediction model in OLS

- Ignoring the clustering of schools
- $Y_i = b_0 + b_1(SES_i) + b_2(Numrisks) + b_3(Female_i) + e_i$

**Coefficients<sup>a</sup>**

Model		Unstandardized Coefficients		Standardized Coefficients	t	Sig.
		B	Std. Error	Beta		
1	(Constant)	56.238	.390		144.115	.000
	wkses	6.116	.316	.372	19.326	.000
	numrisks	-1.047	.312	-.065	-3.351	.001
	female	2.610	.492	.101	5.311	.000

a. Dependent Variable: c4read

37 Compare to Model in HLM (Random Coefficients Model)

- Allow all effects at level one to vary at random
- $Y_{ij} = \beta_{0j} + \beta_{1j}(SES_{ij}) + \beta_{2j}(Numrisks_{ij}) + \beta_{3j}(Female_{ij}) + r_{ij}$
- $\beta_{0j} = \gamma_{00} + u_{0j}$
- $\beta_{1j} = \gamma_{10} + u_{1j}$
- $\beta_{2j} = \gamma_{20} + u_{2j}$
- $\beta_{3j} = \gamma_{30} + u_{3j}$

"Fixed" effects:  $\gamma_{00}, \gamma_{10}, \gamma_{20}, \gamma_{30}$   
 "Random" effects:  $r_{ij}, u_{0j}, u_{1j}, u_{2j}, u_{3j}$

38 Combined Model

- Random Coefficients Model
- Intercept and slopes from level one are allowed to vary at random

Combined Model

$$CAREAD_{ij} = \gamma_{00} + \gamma_{10} * WK * SES_{ij} + \gamma_{20} * NUMRISKS_{ij} + \gamma_{30} * FEMALE_{ij} + u_{0j} + u_{1j} * WK * SES_{ij} + u_{2j} * NUMRISKS_{ij} + u_{3j} * FEMALE_{ij} + r_{ij}$$

Fixed Part  
Random Part

39 Results: Random Coeffs Model

Fixed Effects	Coefficient (SE)	t (df) <sup>a</sup>
Model for the Intercepts ( $\beta_0$ )		
Intercept ( $\gamma_{00}$ )	56.22 (.57)	98.72 **
Model for SES Slopes ( $\beta_1$ )		
Intercept ( $\gamma_{10}$ )	5.28 (.37)	14.09 **
Model for NUMRISKS Slopes ( $\beta_2$ )		
Intercept ( $\gamma_{20}$ )	-.79 (.32)	-2.473 *
Model for FEMALE Slopes ( $\beta_3$ )		
Intercept ( $\gamma_{30}$ )	2.29 (.51)	4.45 **
Random Effects (Var. Components)		
Var. in Intercepts ( $\tau_{00}$ )		30.10 **
Var. in SES Slopes ( $\tau_{11}$ )		2.41 **
Var. in NUMRISKS Slopes ( $\tau_{22}$ )		.79
Var. in FEMALE Slopes ( $\tau_{33}$ )		7.81
Level-one Variance ( $\sigma^2$ )		118.61

<sup>a</sup> DF for all effects is 163. <sup>b</sup> based on Chi-square test  
 (Note: SES is uncentered here)

40 Variance Components

Level-one Variance:  $\sigma^2 = 118.61$

Level-two Variances and Covariances:

$$T = \begin{pmatrix} \tau_{00} & \tau_{01} & \tau_{02} & \tau_{03} \\ & \tau_{11} & \tau_{12} & \tau_{13} \\ & & \tau_{22} & \tau_{23} \\ & & & \tau_{33} \end{pmatrix} = \begin{pmatrix} 30.10 & -2.74 & -3.17 & -9.42 \\ & 2.41 & .28 & 1.71 \\ & & .79 & 2.26 \\ & & & 7.81 \end{pmatrix}$$

41 Some Findings OLS vs. HLM

- Standard errors in the OLS are always smaller
- Variability exists between schools in effects of SES
- There is no variability between schools in effects of Numrisks or Gender
- There is variability between schools in intercepts
  - What do intercepts mean?
  - School's prediction for males with no family risks and with 0 SES!
  - We need to ask ourselves, is 0 SES a reasonable value here? It may not always be the case
    - Outside the range for some schools
    - Some continuous predictors may not have a natural zero

42 OLS versus HLM (con't)

- In the OLS analysis, there is no mechanism for testing whether school variables – such as SchoolSES, or Neighborhood Climate, or public versus private sector, also contribute to understanding reading scores
  - Violates assumption of independence, since each child in same school receives SAME score on these school-level predictors
- HLM Advantages:
  - Centering – to enhance interpretation of intercept
  - Inclusion of context-level predictors – to enhance our understanding of the dependent variable
  - Adjustment for clustering and inflation of type I error

43 Centering options in HLM

- Centering within Contexts (CWC)  $(X_{ij} - \bar{X}_{.j})$ 
  - Group-mean centering of predictors
  - Each group "centered" at its own mean
  - Intercept then becomes the prediction for the group mean (unadjusted group mean), conditional on any other predictors in the model
- Centering at the grand mean (CGM)  $(X_{ij} - \bar{X}_{..})$ 
  - Intercept becomes an "adjusted group mean," as in analysis of covariance
  - Mean adjusted for all other effects at that level
- Uncentered (Raw form)  $(X_{ij})$

44 Centering: Theoretical and Interpretational Concerns

- Choice of centering (or scaling) of level one independent variables for organizational and longitudinal designs depends on your specific needs for the analysis
  - Theory of how, or if, context variables (level-two predictors) affect the outcome of interest
  - Can enhance interpretation of effects (intercepts and slopes)
- Often we are interested in effects of level-one predictors AND their level-two aggregate
  - Q: are effects the SAME at both levels?

45 RC Model, SES as CWC

$$Y_{ij} = \beta_{0j} + \beta_{1j}(SES_{ij} - SES_{.j}) + \beta_{2j}(Numrisks_{ij}) + \beta_{3j}(Female_{ij}) + r_{ij}$$

$$\beta_{0j} = \gamma_{00} + u_{0j}$$

$$\beta_{1j} = \gamma_{10} + u_{1j}$$

$$\beta_{2j} = \gamma_{20} + u_{2j}$$

$$\beta_{3j} = \gamma_{30} + u_{3j}$$

46 Contextual or Conditional Model

$$Y_{ij} = \beta_{0j} + \beta_{1j}(SES_{ij} - SES_{.j}) + \beta_{2j}(Numrisks_{ij}) + \beta_{3j}(Female_{ij}) + r_{ij}$$

$$\beta_{0j} = \gamma_{00} + \gamma_{01}MeanSES_j + \gamma_{02}NBhoodCLIM_j + \gamma_{03}Private_j + u_{0j}$$

$$\beta_{1j} = \gamma_{10} + \gamma_{11}MeanSES_j + \gamma_{12}NBhoodCLIM_j + \gamma_{13}Private_j + u_{1j}$$

$$\beta_{2j} = \gamma_{20}$$

$$\beta_{3j} = \gamma_{30}$$

Note: in earlier runs, models would not converge when random effects were included on the Numrisks and Gender slopes; the variance components for these slopes were not statistically different from zero in RC model.

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Fixed Effects	Random Coeffs Coefficient (SE)	Contextual Coefficient (SE)
Model for the Intercepts ( $\beta_0$ )		
Intercept ( $\gamma_{00}$ )	56.60 (.65) **	56.10 (.62) **
MEANSES ( $\gamma_{01}$ )		6.77 (.95) **
NBHOODCLIM ( $\gamma_{02}$ )		-.37 (.17) *
PRIVATE ( $\gamma_{03}$ )		3.04 (1.05) **
Model for SES Slopes ( $\beta_1$ )		
Intercept ( $\gamma_{10}$ )	4.45 (.43) **	4.52 (.60) **
MEANSES ( $\gamma_{11}$ )		2.08 (.91) *
NBHOODCLIM ( $\gamma_{12}$ )		-.18 (.19)
PRIVATE ( $\gamma_{13}$ )		-3.76(1.14) **
Model for NUMRISKS Slopes ( $\beta_2$ )		
Intercept ( $\gamma_{20}$ )	-.91 (.33) **	-.57 (.31)
Model for FEMALE Slopes ( $\beta_3$ )		
Intercept ( $\gamma_{30}$ )	2.22 (.51) **	2.32 (.47) **
Random Effects (Var. Components)	Variance	Variance
Var. in Intercepts ( $\tau_{00}$ )	46.51 **	15.59 **
Var. in SES Slopes ( $\tau_{11}$ )	4.49 **	.74 *
Var. in NUMRISKS Slopes ( $\tau_{22}$ )	.71	---
Var. in FEMALE Slopes ( $\tau_{33}$ )	7.52	---
Level-one Variance ( $\sigma^2$ )	117.62	120.27

Note: FEML, with SES as CWC

48 Reduction in Intercept and Slope Variance

- Simple calculation for proportion of variance explained by the level-two predictors:
  - $pval(intercept) = (46.51 - 15.59)/46.51 = 66.48\%$
- But, there is still significant variance remaining in the intercepts ( $\tau_{00} = 15.59, p=.000$ )
- SES is also not well-explained by the three level-two predictors ( $\tau_{11} = .74, p=.040$ )
- But, 83.5% of the variance in the SES slope is explained by the three level-two predictors.
- Typical process? Build up rather than down; remove variables that are not predictive; best to have a theoretical rationale in building models and choosing predictors.

49 Other issues...

- Estimation
  - Empirical Bayes estimation for the regression coefficients at level one
  - Generalized Least Squares for the regression coefficients at level two
  - Maximum likelihood (FEML, FIML, ML) or Restricted maximum likelihood (REML) for the variance components
    - REML best if number of level-two units is small

50 Model Fit

- Model Fit (McCoach & Black, 2008)
  - Every model provides a deviance =  $-2LL$ 
    - Poorness of fit, smaller is better
    - Can be used in Chi-square Comparison test (LR test), better test of significance for variance components (REML or FEML) or for tests of fixed effects or multi-parameter tests (FEML)
    - Models must be nested. (Previous two models are not nested)
    - Deviance can be adjusted to yield AIC, BIC (which can be used for non-nested models)
    - AIC = Dev + 2p
    - BIC = Dev +  $p \cdot \ln(n)$

51 For our models on slide 47

- Random Coeffs (FEML)
  - Deviance = 18140.09, 15 parameters
  - AIC =  $18140.09 + 2(15) = 18170.09$
  - BIC =  $18140.09 + 15(\ln(2340)) = 18256.46$
- Contextual Model (FEML)
  - Deviance = 18023.18, 14 parameters
  - AIC =  $18023.18 + 2(14) = 18051.18$
  - BIC =  $18023.18 + 14(\ln(2340)) = 18131.79$

Results: smaller IC is better, both information criteria favor Contextual Model

52 Software

- HLM and MLWIN
  - Comprehensive MLM software
  - Residual diagnostics, graphing features
  - Can check assumptions, adjust model for heterogeneity at level-one
    - Linear, non-linear models
- SPSS
  - Designed in language for repeated measures
  - Primarily for continuous outcomes; newer module for GLMM
- SAS
  - Comprehensive: Some folks feel the language, commands not as user-friendly as HLM
  - Great graphics for residual analyses
- MPLUS
  - Merges SEM with HLM
  - Comprehensive, does some great models, some challenges in learning, data setup
- R
  - Freeware on the web
  - Really productive network of users
- STATA
  - Continuous outcomes and also non-linear

53 Advances and New Directions

- Cross-classified and multiple membership designs
  - [http://131.252.97.79/transfer/ES\\_Pubs/ESVal/multilevel\\_analysis/cross-classified/Cross\\_classified\\_review\\_RR791.pdf](http://131.252.97.79/transfer/ES_Pubs/ESVal/multilevel_analysis/cross-classified/Cross_classified_review_RR791.pdf)
- Multilevel factor analysis
  - Schweig, J. (2014)
    - Using measurement models that address the hierarchical structure of data, such as survey data collected from students across classrooms or schools
- Partially-nested designs
  - IES (2014): <http://ies.ed.gov/ncer/pubs/20142000/pdf/20142000.pdf>
- Multilevel adjustments for spatial dependence
  - Verbitsky Savitz & Raudenbush (2009)
    - Improving measurement of neighborhood social processes
- Power and sample size
  - Optimal Design (freeware) [http://sitemaker.umich.edu/group-based/optimal\\_design\\_software](http://sitemaker.umich.edu/group-based/optimal_design_software)
  - Spybrook, J. (2008) in O'Connell & McCoach (2008)

54 References & Selected Readings

Srivastava, S.N. (2008). Cross-classified random effects. In A.A. O'Connell and D.B. McCoach, (Eds.) (2008). *Multilevel Modeling of Educational Data*. Charlotte, NC: IAP.

Bingenheimer, J.B. & Raudenbush, S.W. (2004). Statistical and substantive inferences in public health: Issues in the application of multilevel models. *Annual Reviews in Public Health*, 25, 53-77.

Enders, C.K. & Diavood, T. (2007). Centering predictor variables in cross-sectional multilevel models: A new look at an old issue. *Psychological Methods*, 12(2), 121-138.

Graham, J.E., Deutsch, A., O'Connell, A.A., Karmarkar, A.M., Granger, C.V., Ottenbacher, K.J. (2013). Inpatient rehabilitation volume and functional outcomes in stroke, lower extremity fracture, and lower extremity joint replacement. *Medical Care*, 51(5), 404-412.

Kawachi, I., & Berkman, L.F. (Eds.). (2003). *Neighborhoods and Health*. New York: Oxford University Press.

Lu, J., & O'Connell, A.A. (2012). Obesity, high-calorie food intake, and academic achievement trends among US school children. *Journal of Educational Research*, 105, 391-403.

McCoach, D.B. & Black, A.C. (2008). Evaluation of Model Fit and Adequacy. In A.A. O'Connell, and D.B. McCoach, D.B. (Eds.). *Multilevel Modeling of Educational Data*. Charlotte, NC: IAP.

O'Connell, A.A., Logan, J., Pentimonti, J., & McCoach, D.B. (2013). Linear and Quadratic Growth Models for Continuous and Dichotomous Outcomes. In Y. Petscher & C. Schatschneider (Eds.), (pp. 125-168). *Applied Quantitative Analysis in the Social Sciences* (IV). Routledge.

O'Connell, A.A., & McCoach, D.B. (Eds.) (2008). *Multilevel Modeling of Educational Data*. Charlotte, NC: Information Age Publishing.

Raudenbush, S.W., & Bryk, A. S. (2002). *Hierarchical Linear Models* (2nd ed.). Thousand Oaks, CA: Sage Publications, Inc.

Schweig, J. (2014). Multilevel factor analysis by model segregation: New applications for robust statistical tests. *JEBIS*, 39(5), 384-427.

Shaw, K.M. & Peek, M.K. (2009). Neighborhood context and cognitive decline in older Mexican Americans. *AJE*, 169(9), 1045-1051.

Verbitsky Ravitz, N., & Raudenbush, S.W. (2009). Exploiting spatial dependence to improve measurement of neighborhood social process. *Sociological Methods*, 39(1), 151-183.

THANK YOU!